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ON THE ASYMPTOTIC THEORY OF THE THREE-DIMENSIONAL FLOW OF A HYPERSONIC STREAM OF RADIATING GAS AROUND A BODY*

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The three-dimensional flow of a hyersonic stream of ideal gas round bodies of arbitrary thickness allowing for radiation at high temperatures is investigated using the method of a thin optically transparent shock layer, which is a generalization of the well-known method of a thin shock (boundary) layer /1/. Using the fundamental property of the gas in the thin shock layer, which expresses the conservation of the ratio of the stream component of vorticity along streamlines to the density of the gas /2,3/, an analytic solution is obtained of the non-linear problem of the flow round a body bounded by a surface of zero total curvature. The distribution of the radiation heat flux to the body is determined. The effect of radiation on the flow of gas is considered, as an example, in the neighbourhood of the plane of symmetry of a conical body at the angle of attack.

The flow of a hypersonic stream of radiating gas round a body for the plane and axisymmetric cases has been studied in numerous papers (see /4,5/ and the bibliography there). Recently the first results of a numerical calculation of the three-dimensional hypersonic flow of a selectively radiating gas mixture over a blunted body were obtained in /6/. Two-dimensional flow round bodies was considered in /7,8/ using the method of a thin shock layer /1/.

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1. Using the method of a thin shock layer consider the three-dimensional flow of a hypersonic stream round a body, when the temperature of the gas downstream of the bow shock is high, and it is necessary to take into account the effect of radiation on the flow pattern and the gas dynamic functions. The gas downstream of the shock is considered to be stable and the optical thickness of the shock layer d_r between the shock and the body is assumed to be small, i.e. $d_r = d/L_r \ll 1$ (d and L_r are, respectively, the characteristic thickness of the shock layer and the mean free path of the radiation). We will use a curvilinear orthogonal system of coordinates $\xi\eta\zeta$ attached to the body, in which surfaces parallel to the body surface act as one set of coordinate surfaces, and the other two are developable surfaces formed by the normals to the body surface along the lines of curvature. Let ξ, ζ be the parameters of the lines of curvature and η the distance from the body along the external normal $\mathbf{n}$. The Lamé parameters have the form

$$H_\xi = \sqrt{g_{11}}(1 - K_1\eta), \quad H_\eta = 1, \quad H_\zeta = \sqrt{g_{22}}(1 - K_2\eta) \tag{1.1}$$

where g_{11}, g_{22} are the coefficients of the first basic quadratic form, and K_1 and K_2 are the principal curvatures. We will represent the body surfaces and the compression shock by the equations

$$\mathbf{r} = \mathbf{r}_b(\xi, \zeta), \quad \mathbf{r}_s = \mathbf{r}_b(\xi, \zeta) + S(\xi, \zeta)\mathbf{n}$$

We will introduce the following notation: u, v, w are components of the velocity vector $\mathbf{V}$ in the directions ξ, η, ζ, p is the pressure of the gas, ρ is the density, T is the temperature, h is the enthalpy, μ is the molecular weight, $\varkappa_*$ is the effective adiabatic exponent of the equilibrium gas, σ is Stefan's constant, and $k_p(p, T)$ is the mean Planck absorption coefficient over a fairly wide range of pressures and temperatures, which is represented by the analytic formula /7/

$$k_p(p, T) = apT^n, \ a, n = \text{const} \tag{1.2}$$

Since in hypersonic flow the Mach number of the oncoming stream is $M_\infty \gg 1$ and the effective adiabatic exponent $(\varkappa_* - 1) \ll 1$, then, taking into account the characteristic values of the pressure and enthalpy downstream of the shock, we define the small parameter ε that chacterizes the ratio of the densities of the compression jump and the thickness of the shock layer by the relations

$$\varepsilon = \frac{\varkappa - 1}{\varkappa + 1} \ll 1, \quad \varkappa = \varkappa_*\left(\rho_\infty V_\infty^2, \frac{1}{2}V_\infty^2\right)$$

The characteristic ratio of the temperatures at the shock is determined by the product $m = \varepsilon M_\infty^2$. It follows from the energy equation that the effect of radiation is important when $m \gg 1$. Thus

$$d = \varepsilon L, \quad d_r = \varepsilon L k_p^*, \quad L k_p^* = L a \rho_\infty V_\infty^2 T_\infty^n m^n \equiv l m^n$$

where L is the characteristic dimension of the body. The condition that the shock layer is optically thin (transparent) has the form

$$d_r = l\varepsilon m^n \to 0 \quad \text{as} \quad \varepsilon \to 0, \ m \to \infty \tag{1.3}$$

2. The use of the method of a thin optically transparent shock layer to solve the gas-dynamic equations taking radiation into account involves passing to limit $\varepsilon \to 0$, $M_\infty \to \infty$, $m \to \infty$, $d_r \to 0$. We will introduce the following independent variables of order unity:

$$\xi^\circ = \xi/L, \ \eta^\circ = \eta/\varepsilon L, \ \zeta^\circ = \zeta/L \tag{2.1}$$

and expansions of the unknown functions

$$\begin{aligned}
&u/V_\infty = u_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots, \quad v/V_\infty = \varepsilon v_1(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots \\
&w/V_\infty = w_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots, \quad p = p_\infty + \rho_\infty V_\infty^2 p_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots \\
&2h/V_\infty^2 = h_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots, \quad \rho/\rho_\infty = \varepsilon^{-1}\rho_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots \\
&T/T_\infty = mT_0(\xi^\circ, \eta^\circ, \zeta^\circ) + \ldots, \quad 2q_r/\rho_\infty V_\infty^3 = q_0(\xi^\circ, \zeta^\circ) + \ldots \\
&\varkappa_*(p, h) = 1 + 2\varepsilon\Delta(p_0, h_0) + \ldots, \quad \mu/\mu_\infty = \mu_0 + \ldots \\
&S/L = \varepsilon S_1(\xi^\circ, \zeta^\circ) + \ldots
\end{aligned} \tag{2.2}$$

where q_r is the radiation heat flux on the surface.

Substituting (1.1), (1.2), (2.1), and (2.2) into the equations of radiation gas-dynamics in the approximation of an optically transparent layer of gas /4,5/, assuming, using (1.3), $m = O[(l\gamma\varepsilon)^{-1/(n+4)}]$ and equating terms of like powers of ε, we obtain in the basic approximation the following equations:

$$\sqrt{g_{11}g_{22}}\,Du + w[u(\sqrt{g_{11}})_\zeta - w(\sqrt{g_{22}})_\xi] = 0 \tag{2.3}$$

$$\rho(K_1u^2 + K_2w^2) = -p_\eta \tag{2.4}$$

$$\sqrt{g_{11}g_{22}}\,Dw - u\left[u\,(\sqrt{g_{11}})_\zeta - w\,(\sqrt{g_{22}})_\xi\right] = 0 \tag{2.5}$$

$$(\rho u\,\sqrt{g_{22}})_\xi + \sqrt{g_{11}g_{22}}\,(\rho v)_\eta + (\rho w\,\sqrt{g_{11}})_\zeta = 0 \tag{2.6}$$

$$\rho Dh + \Gamma p T^{n+4} = 0 \qquad p = \rho h \Delta\,(p,\,h), \quad p\mu = \rho T \tag{2.7}$$

$$D \equiv \frac{u}{\sqrt{g_{11}}}\frac{\partial}{\partial\xi} + v\frac{\partial}{\partial\eta} + \frac{w}{\sqrt{g_{22}}}\frac{\partial}{\partial\zeta} \tag{2.8}$$

$$\Gamma = \gamma d_r m^4 = O\,(1), \quad \gamma = 8\sigma T_\infty{}^4/(\rho_\infty V_\infty{}^3)$$

where the superscripts are omitted and coordinate symbols in subscripts denote partial derivatives, D is the differentiation operator along streamlines, and Γ is a parameter characterizing the effect of radiation on the gas flow.

Below, we use the fact that, when $T \geqslant 10^4$ °K the quantity $\varkappa_*$ is virtually constant /9/, and we assume $\Delta = 1$; the constant $\mu_0 \leqslant 1$ which is determined by the equilibrium state of the gas is assumed constant. At the compression shock, when $\eta = S\,(\xi,\,\zeta)$, we have, from /2, 3/, the conditions

$$u_s = \mathbf{e}_\infty\cdot\mathbf{r}_1, \quad w_s = \mathbf{e}_\infty\cdot\mathbf{r}_2 \tag{2.9}$$

$$v_s = u_s S_\xi/\sqrt{g_{11}} + w_s S_\zeta/\sqrt{g_{22}} - v_1$$

$$p_s = h_s = v_1{}^2, \quad \rho_s = 1, \quad T_s = \mu v_1{}^2$$

$$(\mathbf{r}_1 = (\mathbf{r}_b)_\xi, \quad \mathbf{r}_2 = (\mathbf{r}_b)_\zeta, \quad v_1 = \mathbf{e}_\infty\cdot\mathbf{n}, \quad \mathbf{e}_\infty = \mathbf{V}_\infty/V_\infty)$$

On the surface of the body the condition of impermeability must be satisfied

$$v_b = 0 \tag{2.10}$$

Since the shock layer is thin and the derivatives along the layer are small composed with the derivatives normal to it, we use, when the radiation propagation is taken into account, the widely used /5,7,8/ approximation of a locally one-dimensional plane layer. Then, integrating across the vector divergence layer of the radiation heat flux, and using (1.2) and (2.2), we obtain the following expression for the radiation heat flux reaching the body

$$q = \frac{\Gamma}{2}\int_0^{S(\xi,\,\zeta)} pT^{n+4}\,d\eta$$

Note that in agreement with the general result /2,3/ the flow of radiating gas in the shock layer has the property of conserving along the streamlines the ratio of the stream vorticity component to the gas density. This property follows from Eqs.(2.3), (2.5), and (2.6) which hold for any arbitrary flow in the thin shock layer irrespective of the physico-chemical processes taking place in it (equilibrium, non-equilibrium, chemical reactions, dissociation, ionization, and radiation), and is expressed by the equation

$$D\,[u^2\,(w/u)_\eta/\rho] = 0 \tag{2.11}$$

3. Let us consider the flow of radiating gas over a body with a developed surface form, whose total curvature is $K_1K_2 = 0$ (for instance, suppose $K_1 = 0$). As the coordinates ξ, ζ we select the dimensionless length of the arc along the curvature lines, and obtain $g_{11} = g_{22} = 1$, $g_{12} = 0$. Note that such surfaces may be used to construct bodies of optimal aerodynamic form /10/. Instead of the equation of continuity we use Eq.(2.11). Then instead of Eqs.(2.3) —(2.6) we obtain

$$Du = 0, \quad Dw = 0, \quad D\,[(w/u)_\eta/\rho\,] = 0 \tag{3.1}$$

$$\rho K_2 u^2 = -p_\eta, \quad D \equiv u\frac{\partial}{\partial\xi} + v\frac{\partial}{\partial\eta} + w\frac{\partial}{\partial\zeta} \tag{3.2}$$

To obtain a general solution of system (3.1), (3.2), (2.7), and (2.8) we change, as in /2,3/, to new variables ξ, ψ, θ, where $\psi\,(\xi,\,\eta,\,\zeta)$, $\theta\,(\xi,\,\eta,\,\zeta)$ are constants along streamlines, defined by the equations

$$d\xi/u = d\eta/v = d\zeta/w \tag{3.3}$$

From (3.1), (3.3), (2.8), and (2.7) we obtain

$$u = U\,(\psi,\,\theta), \quad w = W\,(\psi,\,\theta), \quad \zeta = N\,(\psi,\,\theta)\xi + Z\,(\psi,\,\theta) \tag{3.4}$$

$$\psi_\eta N_\psi + \theta_\eta N_\theta = \rho\Omega^{-1}\,(\psi,\,\theta) \tag{3.5}$$

$$(n+4)U\,(\psi,\,\theta)\,h_\xi + Kh^{n+5} = 0 \tag{3.6}$$

$$(K = (n+4)\,\mu^{n+4}\Gamma, \; N = W/U)$$

where K is the similarity parameter that takes radiation into account, and U, W, Z, Ω are arbitrary functions.

Taking into account the conditions at the shock $h = h_s$, when $\xi = \chi(\psi, \theta)$, the solution of Eq.(3.6) has the form

$$h = h_s\{1 + Kh_s^{n+1}[\xi - \chi(\psi, \theta)]/U(\psi, \theta)\}^{-1/(n+4)} \tag{3.7}$$

Then assuming, as in /2,3/, $\psi = W/U$, $\theta = Z$, writing (3.2), (3.3), and (3.5) in the variables ξ, ψ, $\zeta = \psi\xi + \theta$, and integrating, taking (2.9) and (2.10) into account, we obtain

$$p(\xi, \psi, \zeta) = p_s(\xi, \zeta) - K_2 \int_{\Psi(\xi,\zeta)}^{\psi} W^2(\psi', \zeta - \psi'\xi)\,\Omega(\psi', \zeta - \psi'\xi)\, d\psi' \tag{3.8}$$

$$\rho = p/h \tag{3.9}$$

$$v(\xi, \psi, \zeta) = U(\psi, \zeta - \psi\zeta)\left\{\int_{\psi_b}^{\psi} \left[\frac{\psi - \psi'}{\rho}\Omega_{\theta'} - (\rho_\xi + \psi\rho_\xi)\frac{\Omega}{\rho^2}\right] d\psi' - \Omega_b[(\psi_b)_\xi + \psi(\psi_b)_\zeta]/\rho_b\right. \tag{3.10}$$

$$S(\xi, \zeta) = \int_{\psi_b}^{\Psi} \Omega(\psi, \zeta - \psi\xi)\,\rho^{-1}(\xi, \psi, \zeta)\, d\psi \tag{3.11}$$

$$(\Psi(\xi, \zeta) = \psi[\xi, S(\xi, \zeta), \zeta],\ \psi_b(\xi, \zeta) = \psi(\xi, 0, \zeta),$$
$$(\psi_b)_\xi + \psi_b(\psi_b)_\zeta = 0 \quad \text{or} \quad \Omega_b = 0)$$

The abscissa of the point of entry $\chi(\Psi, \lambda)$ of the streamline into the shock layer is the root of the functional equation

$$\Psi = w_s(\chi, \Psi\chi + \lambda)/u_s(\chi, \Psi\chi + \lambda)$$

The functions $F = \{u, w, \Omega\}$, which are constant along the streamlines in the flow field, are expressed by their values at the shock

$$F(\psi, \zeta - \psi\xi) = f_s(\chi, \tau), \quad \chi = \chi(\psi, \zeta - \psi\xi), \quad \tau = \zeta + \psi(\chi - \xi)$$

and, as in /2,3/, $\Omega_s = -[\Psi K_2(\xi, \zeta)]^{-1}$.

Thus, in the basic approximation of the method of a thin optically transparent shock layer we have obtained an analytic solution of the problem of the flow over a body of a radiating hypersonic flow of gas. Formulas (3.4) and (3.7)—(3.11) show that in the basic (Newtonian) approximation the radiation does not affect the pressure distribution and the tangential velocity components, but in calculating the density, the vertical velocity component and the three-dimensional shock layer thickness it is necessary to take radiation into account. These properties were established earlier for plane and axisymmetric flows in the shock layer /5, 7,8/.

Changing to the new independent variables ξ, χ, ζ, we transform the formulas for the shock layer thickness and the radiation heat flux to the final form

$$S(\xi, \zeta) = -\int_0^{\xi} \frac{v_1(\chi, \tau)\, h_s(\chi, \tau)\,[1 + Kh_s^{n+4}(\xi - \chi)/u_s(\chi, \tau)]^{-1/(n+4)}}{p(\xi, \chi, \zeta)\, u_s(\chi, \tau)\,[1 - (\xi - \chi)\,\Psi_\zeta(\chi, \tau)]}\, d\chi \tag{3.12}$$

$$q(\xi, \zeta) = -\frac{K}{2(n+4)} \int_0^{\xi} \frac{v_1(\chi, \tau)\, h_s^{n+5}\,[1 + Kh_s^{n+4}(\xi - \chi)/u_s(\chi, \tau)]^{-(n+5)/(n+4)}}{u_s(\chi, \tau)\,[1 - (\xi - \chi)\,\Psi_\zeta(\chi, \tau)]}\, d\chi$$

4. Generally, calculating the flow using the above formulas involves certain mathematical difficulties due to the complicated functional relations. As an example, consider the flow in the neighbourhood of the plane of symmetry on the windward side of a conical body at an angle of attack α. Suppose that in the Cartesian system of coordinates xyz with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ the plane of symmetry corresponds to $\zeta = z = 0$, and the equation of the section of the body by that plane is $y_b = x \operatorname{tg} \varphi$. The principal radii of curvature of the surface in the plane $z = 0$ are

$$R_1 = K_1^{-1} = \infty, \quad R_2 = K_2^{-1} = R_0\xi \operatorname{tg}\varphi \sec\varphi, \quad \xi = x \sec\varphi$$

In the neighbourhood of the plane of symmetry we have

$$w(\xi, \chi, \zeta) = \zeta w_1(\xi, \chi), \quad S(\xi, \zeta) = S_0(\xi) \tag{4.1}$$
$$\Phi(\xi, \chi, \zeta) = \Phi_0(\xi, \chi), \quad \Phi = \{u, v, p, \rho, h, q\}$$
$$\mathbf{r}_1 = \mathbf{i}\cos\varphi + \mathbf{j}\sin\varphi, \quad \mathbf{r}_2 = \mathbf{k} + (\mathbf{i}\sin\varphi - \mathbf{j}\cos\varphi)\,\zeta K_2$$
$$\mathbf{n} = -\mathbf{i}\sin\varphi + \mathbf{j}\cos\varphi + \mathbf{k}\zeta K_2, \quad \mathbf{e}_\infty = \mathbf{i}\cos\alpha - \mathbf{j}\sin\alpha$$

which is accurate to terms of order ζ^2. Taking into account (4.1) and (2.9) we obtain from formulas (3.12) the dependence of the compressed layer thickness and the heat flux from the universal coordinate

$$S_0(X) = \frac{\operatorname{tg}(\varphi + \alpha)}{K} \int_0^X \frac{(1 + X - \chi)^{-1/(n+4)}}{(1 - C)\chi + CX}\, \chi\, d\chi \tag{4.2}$$

$$q_0(X) = \frac{1}{2(n+4)} \int_0^X \frac{(1+X-\chi)^{-(n+5)/(n+4)}}{(1-C)\chi + CX} \chi \, d\chi \tag{4.3}$$

$$(C = R_0^{-1} \operatorname{tg}(\varphi+\alpha) \operatorname{ctg}\varphi, \; X = K\xi \sin^{2(n+4)}(\varphi+\alpha) \sec(\varphi+\alpha))$$

where C is a parameter. The integrals (4.2) and (4.3) are convergent, since their integrands have singularities outside the region of integration.

In a number of cases formulas (4.2) and (4.3) convert to results obtained earlier by methods applicable only to flows of special form. Thus when $K = 0, R_0 = 1$ we obtain a formula for the angle of inclination of the attached shock on the windward side of a circular cone at the angle of attack (without radiation)

$$\theta_+ = \varphi + \frac{\varkappa - 1}{\varkappa + 1} \frac{\operatorname{tg}(\varphi+\alpha)}{t^2} [(1+t)\ln(1+t) - t] \tag{4.4}$$

$$t = \frac{\sin\alpha}{\sin\varphi \cos(\varphi+\alpha)}$$

which is the same as the result in /11/ for $M_\infty = \infty$. When $\alpha = 0, C = 0\ (R_0 = \infty)$ and $C = 1\ (R_0 = 1)$ formulas (4.2) and (4.3) yield S_0 and q_0, corresponding to the flow over a wedge and cone of a hypersonic stream of radiating gas.

The results presented in this paper relate to the investigation of essentially three-dimensional effects in a hypersonic flow of radiating gas. Radiation leads, unlike (4.4), to distortion of the bow compression shock, and the field of flow over a conical body no longer has conical properties, and must be investigated using three-dimensional equations. It follows from (4.2) and (4.3) that the upper estimate of the radiation effect corresponds to $n = 0$. Curves of the function $\Pi(X) = KS_0(X) \cdot \operatorname{ctg}(\varphi+\alpha)$ (the continuous lines) and $q_0(X)$ (the dashed lines) are shown for this case in Fig.1, where the parameter C has values of 0, 0.1, 0.5, and 1, relating to curves 1, 2, 3, and 4. These data indicate that a reduction in the radius of curvature of the body cross section for fixed φ, α or an increase in the angle of attack for constant R_0, φ results in a reduction in the shock-layer thickness and in the radiation heat flux to the surface.

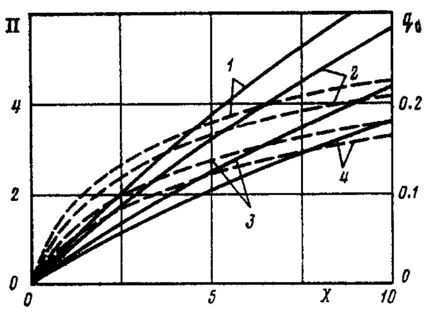

Fig.1

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